

37. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2$$

where v_0 is the initial velocity and a is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2} \frac{v_0^2}{a} = v_0 t_r - \frac{1}{2} \frac{v_0^2}{a} .$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2} \frac{v_{01}^2}{a}$$

and

$$x_2 = v_{02} t_r - \frac{1}{2} \frac{v_{02}^2}{a} .$$

Solving these equations simultaneously for t_r and a we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02} (v_{02} - v_{01})}$$

and

$$a = -\frac{1}{2} \frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2} .$$

Substituting $x_1 = 56.7$ m, $v_{01} = 80.5$ km/h = 22.4 m/s, $x_2 = 24.4$ m and $v_{02} = 48.3$ km/h = 13.4 m/s, we find

$$t_r = \frac{13.4^2(56.7) - 22.4^2(24.4)}{(22.4)(13.4)(13.4 - 22.4)} = 0.74 \text{ s}$$

and

$$a = -\frac{1}{2} \frac{(13.4)22.4^2 - (22.4)13.4^2}{(13.4)(56.7) - (22.4)(24.4)} = -6.2 \text{ m/s}^2 .$$

The *magnitude* of the deceleration is therefore 6.2 m/s². Although rounded off values are displayed in the above substitutions, what we have input into our calculators are the “exact” values (such as $v_{02} = \frac{161}{12}$ m/s).